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Modeling and Experimental Validation of the Effective Bulk Modulus of a Mixture of Hydraulic Oil and Air

It is well known that the presence of entrained air bubbles in hydraulic oil can significantly reduce the effective bulk modulus of hydraulic oil. The effective bulk modulus of a mixture of oil and air as pressure changes is considerably different than when the oil and air are not mixed. Theoretical models have been proposed in the literature to simulate the pressure sensitivity of the effective bulk modulus of this mixture. However, limited amounts of experimental data are available to prove the validity of the models under various operating conditions. The major factors that affect pressure sensitivity of the effective bulk modulus of the mixture are the amount of air bubbles, their size and the distribution, and rate of compression of the mixture. An experimental apparatus was designed to investigate the effect of these variables on the effective bulk modulus of the mixture. The experimental results were compared with existing theoretical models, and it was found that the theoretical models only matched the experimental data under specific conditions. The purpose of this paper is to specify the conditions in which the current theoretical models can be used to represent the real behavior of the pressure sensitivity of the effective bulk modulus of the mixture. Additionally, a new theoretical model is proposed for situations where the current models fail to truly represent the experimental data. [DOI: 10.1115/1.4027173]

Introduction

Bulk modulus is a fundamental and inherent property of fluids which expresses the change in density of the fluid as external pressure is applied to the fluid. It shows both the stiffness of the system and the speed of transmission of pressure waves. Therefore, the stability of servo-hydraulic systems and efficiency of hydraulic systems are affected by the value of fluid bulk modulus [1].

An extensive review of the fundamental concepts, definitions, and experimental techniques for the measurement of fluid bulk modulus was presented by the authors in Ref. [2]. It was pointed out that at any desired temperature and pressure, there are four different values of bulk modulus with considerable differences between them. These four different bulk moduli (which relates to the thermodynamic condition as well as the mathematical condition) are: isothermal secant, isothermal tangent, adiabatic secant, and adiabatic tangent. It is very critical to fully understand the various definitions of fluid bulk modulus and use them correctly, because how the measurement is made can influence the actual bulk modulus value.

The effect of pressure and temperature on the bulk modulus of hydraulic oil in the absence of air has been well studied and some semiempirical equations have been provided which can predict the variation of the hydraulic oil bulk modulus with pressure and temperature with an acceptable accuracy of about $\pm 5\%$. However, the presence of air in the hydraulic oil significantly reduces the fluid bulk modulus and these empirical equations are no longer valid.

Assuming the oil is inside a rigid container, the theoretical models to find the effective fluid bulk modulus in the presence of the mixture of oil and air has been derived by various researchers. A summary of the theoretical models was presented in Ref. [3]. A

comparison of these models in the low pressure region, where the effect of air on the fluid bulk modulus is significant, has indicated that several issues do need to be addressed. A “compression only” model has been developed in some studies where only the volumetric compression of air was considered. Experimental verification of the “compression only” model was presented in Refs. [4] and [5]. In Ref. [4], the air was added as a free pocket (lumped air) at the top of the oil and the maximum amount of air added was 1%. In Ref. [5], the free air content was varied in a range up to 0.5%. The air was injected through a valve, but the air distribution was unknown or at least not mentioned in the paper. In both studies, the applicability of the “compression only” model was verified successfully in the range of their experimental limitations. However, in none of the above mentioned studies, the exact conditions under which the “compression only” model would match the experimental results were mentioned. It is therefore very important to define the conditions in which the “compression only” model can be used.

There is also another condition that exists which has not been extensively studied and this is the situation where air both compresses and dissolves into solution. Very limited literature exists in which the effect of air dissolving into solution has been incorporated into the model of the effective bulk modulus. A “compression and dissolve” model introduced in Ref. [6] did not match well with the experimental results at the lower pressures (up to about 10 MPa). A “compression and dissolve” model, which was developed in Ref. [7] and is used in AMESIM simulation software, was investigated experimentally in Ref. [8], and it was found that the model underestimated the amount of air that was being dissolved in the oil, especially at higher amounts of air (more than 2%). Later in Ref. [9], the authors pointed out that the bulk modulus definition should only be applied to a control volume with a constant mass. Therefore, in modeling the effective bulk modulus of a mixture of oil and air, the constraint was that the mass of air must remain constant as pressure changes. This fact was overlooked in the previous literature and led to the presence of a discontinuity in the previous “compression and dissolve”

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models. A new “compression and dissolve” model was proposed by the authors in Ref. [9]. However, a comparison of the new proposed model with the experimental results showed that there was still poor agreement between the measured values and the new model which indicated that the model still needed to be improved.

In the study presented here, the proposed model in Ref. [9] was revisited and an updated “compression and dissolve” model was developed by introducing a new definition for P_C and a new parameter of $(X_0)_C$. An experimental apparatus was built which was capable of compressing the test fluid at different rates. The test fluid could be used in the form of pure oil, oil and lumped air, or oil and distributed air. The fluid bulk modulus was calculated based on the experimental results, and the proposed theoretical models were then verified with the calculated fluid bulk modulus.

“Compression Only” Model

Referring to Ref. [3], for models in which only the volumetric compression of air was considered, the following theoretical model was recommended:

$$K_{ec} = \frac{V_1(P, T) + V_{gc}(P, T)}{\frac{V_1(P, T)}{K_1(P, T)} + \frac{1}{K_g} V_{gc}(P, T)} \quad (1)$$

where

$$\begin{aligned} K_g &= nP \\ V_{gc}(P, T) &= \left(\frac{P_0}{P}\right)^{\frac{1}{n}} \frac{T}{T_0} X_0 \\ K_1(P, T) &= K_1(P_0, T) + m(P - P_0) \end{aligned}$$

and

$$V_1(P, T) = V_1(P_0, T) \left(1 + \frac{m}{K_1(P_0, T)} (P - P_0)\right)^{-\frac{1}{m}}$$

This model was derived after comparing and modifying the models in Refs. [10–12]. Since parameters $K_1(P_0, T)$, n , X_0 , and m are fixed when the temperature is constant, the effective bulk modulus is only a function of pressure. The values of the above mentioned parameters needed to be identified from measured data.

Proposed New “Compression and Dissolve” Model

A new “compression and dissolve” model, which relates the effective bulk modulus of a mixture of oil and air to the volumetric variations of the air due to both compression and dissolving of air in the oil and also the change in volume and bulk modulus of the pure oil to the pressure and temperature, is developed in Appendix A and given by

$$\left\{ \begin{aligned} K_{ecd} &= \frac{V_1(P, T) + V_{gcd}(P, T)}{\frac{V_1(P, T)}{K_1(P, T)} + \frac{1}{K_{g1}} V_{gcd}(P, T)} & P \leq P_C \\ K_{ecd} &= \frac{V_1(P, T) + \left(\frac{P_0}{P}\right)^{\frac{1}{n_2}} \frac{T}{T_0} (X_0)_C}{\frac{V_1(P, T)}{K_1(P, T)} + \frac{1}{K_{g2}} \left(\frac{P_0}{P}\right)^{\frac{1}{n_2}} \frac{T}{T_0} (X_0)_C} & P > P_C \end{aligned} \right. \quad (2)$$

where

$$K_{g1} = n_1 P$$

$$K_{g2} = n_2 P$$

$$\begin{aligned} V_{gcd}(P, T) &= \left(\frac{P_0}{P}\right)^{\frac{1}{n_1}} \frac{T}{T_0} X_0 \left(\left(\frac{P_C - P}{P_C - P_0}\right) \left(1 - \frac{(X_0)_C}{X_0}\right) + \frac{(X_0)_C}{X_0} \right) \\ K_1(P, T) &= K_1(P_0, T) + m(P - P_0) \end{aligned}$$

and

$$V_1(P, T) = V_1(P_0, T) \left(1 + \frac{m}{K_1(P_0, T)} (P - P_0)\right)^{-\frac{1}{m}}$$

Since parameters $K_1(P_0, T)$, n_1 , n_2 , X_0 , $(X_0)_C$, P_C , and m are fixed when the temperature is constant, the effective bulk modulus is only a function of pressure. The values of these parameters needed to be identified from measured data. The subscript “C” defines a critical point in which compression only occurs due to air being completely dissolved into solution or the local area surrounding the air bubbles becomes saturated and no more dissolving can occur. This is described in greater detail in Appendix A.

Experimental Apparatus and Procedure

Among the different methods of measuring effective fluid bulk modulus, the “volume change method” was chosen in this study because of its higher accuracy [5,13]. In this method, the volume of a cylinder containing the fluid is changed and the bulk modulus is determined from this change. Reference [5] compared the accuracy of the “volume change method” with the “mass change method” and the “sound speed method.” In the “mass change method,” the volume of a cylinder containing the fluid was constant (constant control volume) and some fluid (with a controlled flow rate) was fed into the cylinder. In the “sound speed method,” the measurement of the speed of sound was used to calculate the effective bulk modulus. Reference [4] reported that the “volume change method” was 5 times more accurate than the “mass change method” and 25 times more accurate than the “sound speed method.” The higher accuracy of the “volume change method” was related to the accuracy of the pressure and position sensors that are used. The inferior accuracy of the “mass change method” and the “sound speed method” was due to the lower accuracy of the flow rate measurements and the sampling rate of the data acquisition system, respectively. It should be noted that these limitations in the accuracy of the “sound speed method” and “mass change method” may overcome by utilizing more accurate flow rate measurements and higher sampling rate of data acquisition system.

Schematic diagrams of the apparatus are shown in Fig. 1. Experiments were essentially carried out in three different cases. Figure 1(a) shows a schematic diagram of the apparatus in the baseline and “lumped air” cases. With some modifications to this apparatus, the third case of the experimental investigation, which was called the “distributed air” case, was carried out where air was continuously injected into the system. Figure 1(b) shows the schematic diagram of the apparatus for this third case.

The first case, which was called the “baseline” case, involved the measurements of the tangent bulk modulus of the degassed test oil at different volume change rates. In this case, steps were taken to ensure that the apparatus interior was free of any trapped air. In order to make sure that any residual air bubbles were removed from the system, a partial vacuum was also applied to the system using a vacuum pump (10). A throttle valve (9) was used to control the amount of vacuum pressure for the degassing of the oil. Degassing was continued until no air bubbles were observed through the transparent tube (8).

After degassing the oil, the needle valve (7) was closed and the degassed oil was compressed by actuating the hydraulic cylinder (3) which was mechanically linked to the hydraulic cylinder (4). The volume change rate was controlled by the servo valve (2)

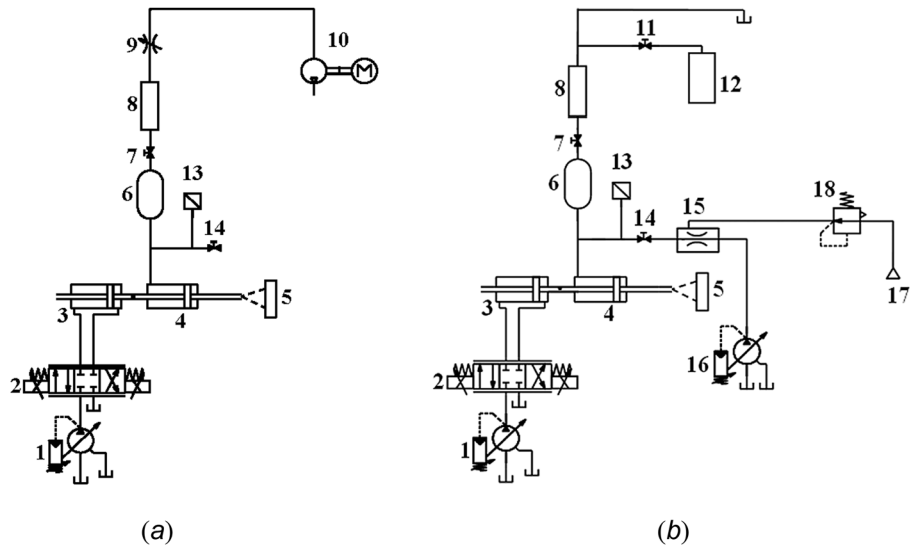


Fig. 1 Schematic diagrams (a) baseline and lumped air cases and (b) distributed air case of the bulk modulus tester. (1) Pressure compensated variable displacement pump, (2) pressure control servo valve, (3) double acting double rod end hydraulic cylinder, (4) double acting double rod end hydraulic cylinder, (5) displacement sensor (MicroTrak II-SA), (6) testing vessel, (7) needle valve, (8) transparent tube, (9) variable throttle, (10) vacuum pump, (11) needle valve, (12) pycnometer, (13) pressure transducer, (14) needle valve, (15) venturi orifice, (16) pressure compensated variable displacement pump, (17) compressed air source, and (18) pneumatic pressure regulator.

through a closed-loop position feedback control system. At the same time, that the oil was compressed, the pressure readings of pressure transducer (13) were taken, and the change in the displacement of the cylinder was measured by a displacement sensor (5). A pressure/displacement curve was recorded for each rate of compression and used for calculating the tangent bulk modulus.

The second case of the experiments was started right after the baseline case. In this case, which was called the “lumped air” case, a large air bubble was created at the top of the oil column by removing a specific amount of oil from the system by opening the needle valve (14). Since the needle valve (7) was also opened at the same time, the same amount of oil, which was removed by opening the needle valve (14), was replaced by the air at the top of the oil column. After a desired amount of lumped air was obtained at the top of the oil column, both needle valves (14) and (7) were closed and the pressure/displacement curve was recorded at different volume change rates.

The third case of the experimental investigation consisted of measuring the tangent bulk modulus of a mixture of oil and air, where air was distributed in the oil in the form of small air bubbles. Figure 1(b) shows the schematic diagram of the experimental setup at this case. A venturi orifice (15) was used to inject and mix air with the oil. It was found that the longer the time interval that air was added, the greater the amount of air that become dispersed in the oil. Unlike the previous case, small air bubbles with different sizes were generated. Before air was injected into the system, the hydraulic cylinder (4) was first completely moved to the left side. After the air was distributed in the circuit, the hydraulic cylinder (4) was slowly moved back to the right side. Therefore, the left chamber of the hydraulic cylinder (4) was also filled with the mixture. At this time, the needle valves (14) and (7) were closed and the mixture was compressed at a specified volume change rate. The pressure/displacement curve was recorded at different volume change rates and used for the bulk modulus calculations.

The volume of the testing vessel, including extra volume added due to the fittings, was $2.133 \times 10^6 \text{ mm}^3$. All the measurements were taken at a temperature of $24 \pm 1^\circ\text{C}$. A couple of lip type piston seals were added in the hydraulic test cylinder (4) in order to

prevent leakage from the test chamber into the cylinder. Leakage tests were performed at a maximum pressure of 6.9 MPa for a duration of 3.5 min and no measurable amount of leakage was observed. The errors in estimating the bulk modulus caused by the deformation of the testing vessel and cylinder were estimated and removed from the final test results.

Equation (3) was used to estimate the maximum uncertainty of the calculated bulk modulus in the experimental setup under study and is given by [13]

$$\varepsilon_{\max} = \pm \left(\varepsilon_P \frac{P_{\max}}{P'} + \varepsilon_{V_0} \frac{V_{\max}}{V'_0} + \varepsilon_x \frac{x_{\max}}{x'} \right) \quad (3)$$

where V_0 is the initial volume of the fluid inside the cylinder at atmospheric pressure and x represents the piston movement when the applied force to the piston increases. The primed values of P , V_0 , and x represent the measured values of these parameters, while the unprimed values of these parameters show the true values of these parameters which are unknown. The parameters ε_P , ε_{V_0} , and ε_x denote the uncertainty of the pressure, volume, and displacement sensors, respectively and $P_{\max}$, $V_{\max}$, and $x_{\max}$ are the maximum measurement ranges of these sensors. It is usually preferred to express the uncertainty values in a dimensionless form. A dimensionless uncertainty in the measurements was obtained by dividing the calculated uncertainty to the maximum measurement range of each sensor. Table 1 lists the range and the dimensionless uncertainty of each sensor.

From the information in Table 1, the maximum uncertainty measurement of the fluid bulk modulus under study was

Table 1 Specification of sensors

Sensor type	Range	ε
Validyne DP15TL pressure transducer	0–6.9 MPa	± 0.0059
Microtrak II stand-alone laser sensor	0–100 mm	± 0.0028
Graduated cylinder	0–2000 ml	± 0.005

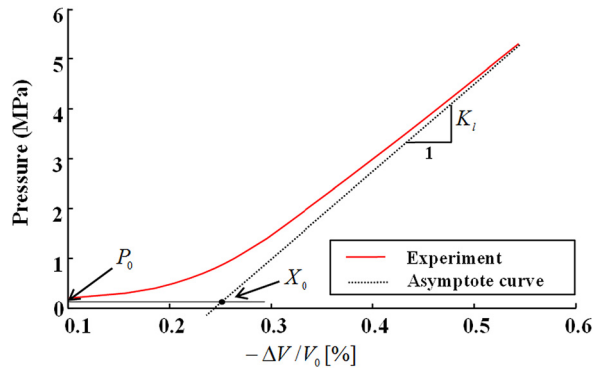


Fig. 2 Calculating X_0 from the experimental results

approximated and is represented as an error bar on appropriate plots of pressure versus fluid bulk modulus.

The value of X_0 needed to be calculated at both the second and third cases. Figure 2 shows a method of estimating X_0 which has been used and explained in Refs. [4], [5], and [14]. In this method, X_0 was estimated from the change in the volume versus pressure curve. By drawing an asymptotic line from the maximum slope to the abscissa, X_0 was determined.

It should be noted that in the second case, the amount of air was known and controlled and hence repeatability tests were possible. However, the air content in the third case could only be ascertained after the test was completed and it was not possible to do a standard repeatability test other than to compare results that had similar air content. It was observed that the amount of air was slightly different in each test (3.5%, 3.7%, and 3.8%), and as such, repeatability could not be formally established. However, the trends (and magnitudes) for these cases were very similar and given the repeatability of the tests with no air, the authors were confident in the reliability of these tests.

Experimental Results of the Baseline Case

The main objective of the baseline case was to obtain the bulk modulus of the test oil (Esso Nuto H68) in the experimental system (which may contain small amounts of air) and compare it to existing isothermal and adiabatic values found in the literature (more details are provided in Appendix B). It is difficult to obtain either pure isothermal or pure adiabatic conditions, but it can be assumed that a very rapid test would give nearly adiabatic results and a very slow test would give nearly isothermal results. For different rates of volume change, it may be reasonable to assume that the bulk modulus value would lie somewhere between values for isothermal and adiabatic conditions.

Figure 3 shows a typical result of pressure versus bulk modulus (at a predetermined volume change rate (%/s)) for the baseline case of the fluid used in this study compared to the expected bulk modulus of pure oil. Also shown as an inset in Fig. 3 is a pressure versus time plot of the test oil as a result of the change in volume during the test.

According to the literature, the bulk modulus of any pure oil has a linear relationship with pressure [15,16]. However, experimental results of Fig. 3 do not show a linear relationship. The bulk modulus changes nonlinearly up to a pressure of 1.5 MPa, followed by a relationship that is nearly linear. Note that the effect of deformation of the testing vessel and cylinder was estimated and subtracted off from the experimental results. Therefore, the only factor that could have contributed to this initial nonlinear behavior is the presence of small amounts of air. Despite efforts to take all the air out of the system, some air must have remained trapped in the container resulting in this nonlinear behaviour. The presence of these small amounts of air was also a concern of Hayward [15], where he suggested that an initial pressure of at least 1 MPa be used when determining the bulk modulus of pure oil.

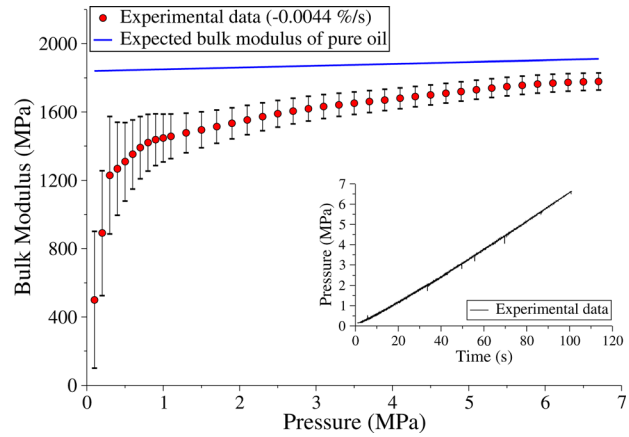


Fig. 3 A typical example of bulk modulus measurement in the baseline case ($-0.0044\%/s$)

In order to account for this small amount of air and to try model this characteristic, the effective bulk modulus is defined as

$$\frac{1}{K_{\text{eff}}} = \frac{1}{K_1(P, T)} + \beta \frac{1}{K_g} \quad (4)$$

where

$$K_1(P, T) = K_1(P_0, T) + m(P - P_0) \\ K_g = P$$

and

$$\beta = \frac{V_g}{V_g + V_l}$$

Since the initial volume of air was quite small, the volumetric variation of air would not be significant and it could be assumed that the term β is a constant. Parameter m , which shows the slope of the expected bulk modulus of pure oil, is a function of temperature. Since temperature was maintained at a constant value of $24 \pm 1^\circ\text{C}$, this parameter was expected to be constant during the experiments. The value of m is well known for mineral hydraulic oils and thus did not need to be estimated. Referring to Appendix B, $m = 10.4$ for the test oil used in the experiments. Therefore, the only parameters which were required to be estimated were $K_1(P_0, T)$ and β . In order to determine these two parameters, a least squares approach similar to Ref. [6] was used. Here, matrix Z is defined as

$$Z = \begin{pmatrix} K_1(P_0, T) \\ \beta \end{pmatrix} \quad (5)$$

The function $F(Z)$ is defined as the sum of the squared errors between the inverse of the experimental bulk modulus and the predicted bulk modulus as

$$F(Z) = \sum_{i=1}^N \left(\frac{1}{\hat{K}_{\text{eff}}} - \frac{1}{K_{\text{eff}}} \right)^2 \quad (6)$$

where $\hat{K}_{\text{eff}}$ is the effective bulk modulus measured experimentally in a hydraulic system subject to pressures P_i , $i = 1, 2, \dots, N$. The boundary condition of matrix Z is determined as follows:

$$Z_l \leq Z \leq Z_u \\ Z_l = \begin{pmatrix} 1534 \\ 0 \end{pmatrix}, \quad Z_u = \begin{pmatrix} 1972 \\ 0.5 \end{pmatrix} \quad (7)$$

The lower and upper limits of $K_I(P_0, T)$ were determined by the predicted values from Appendix B. The lower limit of β was set to zero and the upper limit, which represents the highest possible amount of air, was set to 0.5%.

The objective of the least squares method was to determine a value of Z so that its substitution into Eq. (6) would minimize $F(Z)$. The least squares modeling error was determined by calculating the average error E which is given in Ref. [6] as

$$E = \sqrt{\frac{1}{N} \sum_{i=1}^N \left(\frac{1}{\hat{K}_{\text{eff}}} - \frac{1}{K_{\text{eff}}} \right)^2} \quad (8)$$

The MATLAB built in function “lsqcurvefit” was employed to find model parameters and errors with respect to the experimental data. Once parameters $K_I(P_0, T)$ and β were determined using this least squares method, a theoretical curve based on Eq. (4) was determined. This is included in the figures of the different testing conditions which appear in “Isothermal and Adiabatic Tangent Bulk Modulus of the Test Oil” and “Tangent Bulk Modulus of the Test Oil for Other Volume Change Rates” sections.

Isothermal and Adiabatic Tangent Bulk Modulus of the Test Oil. In order to determine the isothermal and adiabatic tangent bulk modulus of the test oil, the results of the slowest and fastest volume change rate were chosen. Figure 4 shows the experimental results of the baseline case and the theoretical results based on the nonlinear least squares curve fit of the experimental data when the volume change rate was $-0.0006\%/s$. An average modeling error of $E = 76$ MPa was calculated.

The estimated parameters were found as $K_I(P_0, T) = 1603$ MPa and $\beta = 0.0002$. From Appendix B, the isothermal bulk modulus value of 1615 ± 81 MPa was predicted for the test oil at atmospheric pressure and a temperature of 24°C . It was evident that the estimated value of 1603 MPa was within the predicted range which confirmed the validity of the baseline case. The estimation of $\beta = 0.0002$ showed that a very small amount of air (0.02%) was present in the system.

Figure 5 shows the experimental results of the baseline case and the theoretical results based on the nonlinear least squares curve fit of the experimental data when the volume change rate was $-0.216\%/s$. For this case, an average modeling error of $E = 158$ MPa was calculated.

The estimated parameters were found as $K_I(P_0, T) = 1972$ MPa and $\beta = 0.0002$. From Appendix B, an adiabatic tangent bulk modulus value of 1878 ± 94 MPa was predicted for the test oil at atmospheric pressure and a temperature of 24°C . The estimated value of 1972 MPa was within the predicted range which again

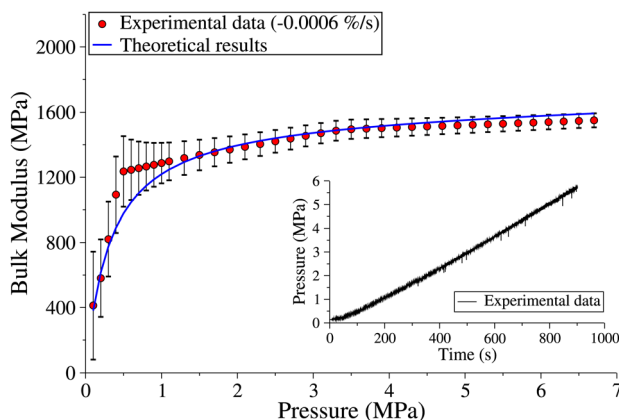


Fig. 4 Nonlinear least square curve fit of the experimental data in the baseline case when the volume change rate was $-0.0006\%/s$

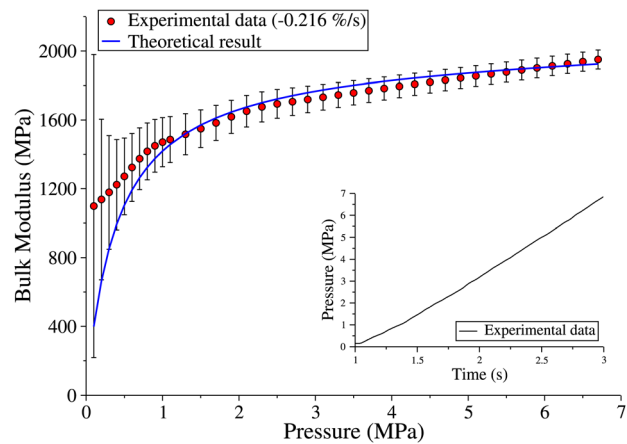


Fig. 5 Nonlinear least squares curve fit of the experimental data in the baseline case when the volume change rate was $-0.216\%/s$

Table 2 $K_I(P_0, T)$ versus the oil compression time obtained from the baseline case

Oil compression time (s)	$K_I(P_0, T)$ (MPa)
2	1972
10	1966
100	1841
400	1713
900	1603

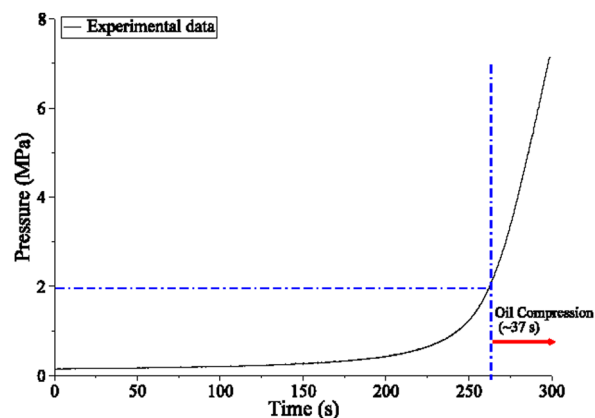


Fig. 6 An example of a change in the pressure when air is added to the oil and compressed

confirmed the validity of the baseline case. The same value of $\beta = 0.0002$ was estimated again which showed that regardless of the volume change rate, its value was repeatable.

Tangent Bulk Modulus of the Test Oil for Other Volume Change Rates. In “Isothermal and Adiabatic Tangent Bulk Modulus of the Test Oil” section, the isothermal and adiabatic tangent bulk moduli of the test oil were estimated and it was found that the estimated results were in a good agreement with known values from the literature. For other rates of volume change, the bulk modulus values were estimated using the nonlinear least squares approach.

The values of $K_I(P_0, T)$ versus oil compression time are summarized in Table 2. These values will be used in “Experimental Results of the Lumped Air Case” and “Experimental Results of the Distributed Air Case” sections to determine the lower and upper limits of the test oil bulk modulus which should be used in the least squares method. As illustrated in Fig. 6, when there is a mixture of oil and air, the oil compression time can be estimated

from the measured pressure/time plot. In Fig. 6, although the mixture of oil and air is being compressed for a duration of about 300 s, a majority of time is spent to compress the air and the oil is only compressed for a duration of 37 s approximately. The oil compression is started from the lowest point where the straight line departs from the curved line and in the example of Fig. 6, this happens approximately after the pressure reaches 2 MPa. As it will be discussed in the next cases, the bulk modulus of the pure oil is needed when theoretical models have to be compared with the experimental results. Therefore, the compression time, which is found from the measured pressure/time curves, is used to estimate the bulk modulus of the pure oil. The approximate value of the oil compression time was obtained in subsequent tests from the pressure/time curves and was related to the corresponding values of $K_I(P_0, T)$.

For the baseline case, however, the pressure/time curve was mostly linear and therefore the oil started to be compressed right from the start. For these tests, the approximate oil compression time is the total time of the test. These results are shown in Table 2 and these values are used to find the lower and upper limits of the test oil bulk modulus when air is added to the system.

Experimental Results of the Lumped Air Case

In this case, air was added in the form of “lumped air” to the top of the oil column. Since the only contact between air and oil was at the top of the oil column and the contact area was also very small, the possibility of air dissolving into the oil was minimal and assumed insignificant. Therefore, as a first fit, the “compression only” model was chosen for comparison with the experimental results. If the “compression only” model fails to accurately model the experimental data, the “compression and dissolve” model can be considered as an alternative model.

The same nonlinear least squares method explained previously was also employed here. Again parameter m is constant and is equal to 10.4. Also parameter X_0 was estimated from the change in the volume versus pressure curve. Therefore, the only two unknown parameters that needed to be estimated using the least squares method were n and $K_I(P_0, T)$. Following the same procedure as used earlier, the matrix Z is now defined as

$$Z = \begin{pmatrix} n \\ K_I(P_0, T) \end{pmatrix} \quad (9)$$

Parameter n is the polytropic index of air which can theoretically range from 1 to 1.4, depending on the heat exchange rate between the air bubbles and the surrounding oil. For the isothermal and adiabatic compression of air, n equals to 1 and 1.4, respectively. Parameter $K_I(P_0, T)$ which is the tangent bulk modulus of oil at atmospheric pressure, can range from 1534 to 1972 MPa, depending on the volume change rate of oil. This range can be narrowed down further by analyzing the measured pressure/time curve of each experiment and obtaining the oil compression time as explained earlier. The resulting oil compression time can then be compared with the bulk modulus values given in Table 2 and a new range for $K_I(P_0, T)$ can be obtained. For example, if the oil compression time was 40 s, a new range of $K_I(P_0, T)$ between 1841 and 1966 MPa would be chosen. This method was repeated for each measurement and a new range of $K_I(P_0, T)$ was found.

In “Experimental Results With 1%, 3%, and 5% of Lumped Air”, the experimental data and the theoretical best fit of each experiment are plotted using the results of the least squares method. The estimated values of n and $K_I(P_0, T)$ are also given for each experiment. For comparison purposes, another two sets of theoretical results are plotted in each figure for the two extreme cases of $n = 1$ and $n = 1.4$. A plot of change in pressure with time is also insetted.

Experimental Results With 1%, 3%, and 5% of Lumped Air. Figures 7–9 depict the experimental and theoretical bulk modulus changes as a function of pressure for 1%, 3%, and 5% of

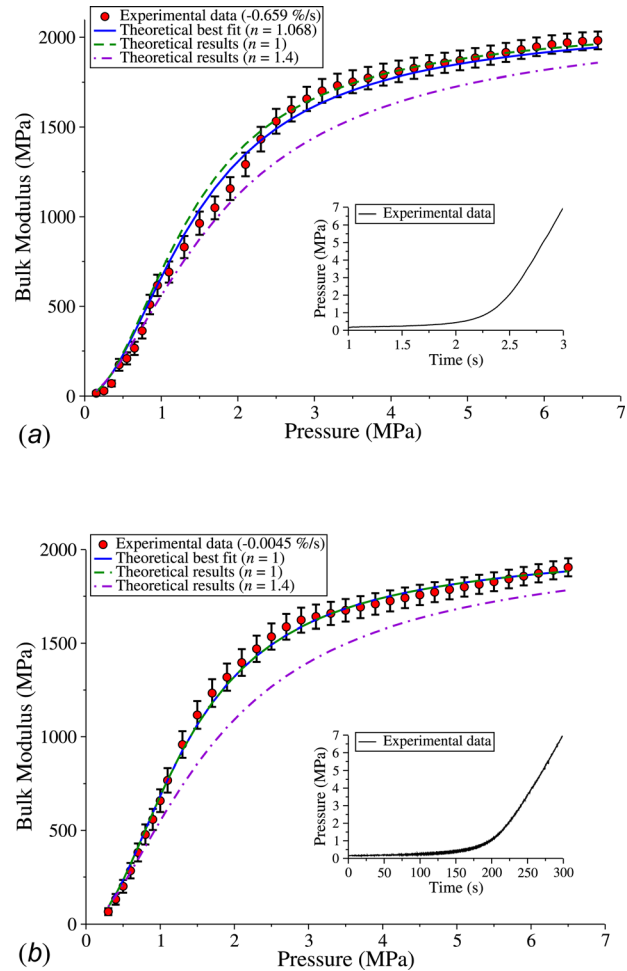


Fig. 7 (a) and (b) Nonlinear least squares curve fit of the experimental data with 1% of lumped air: (a) $K_I(P_0, T) = 1972$ MPa, $n = 1.068$, and $E = 50$ MPa; (b) $K_I(P_0, T) = 1898$ MPa, $n = 1$, and $E = 27$ MPa.

lumped air, respectively. The volume change rate is also noted in each figure. A summary of the results is given in Table 3.

In general, the value of “ n ” is close to the isothermal compression of air value ($n = 1$) for slow volume change rates and tends toward the adiabatic compression of air value ($n = 1.4$) for rapid volume change rates. However, this trend is different when $X_0 = 1\%$. For this case, (Figure 7(a)), the experimental results follow the theoretical curve for $n = 1.4$ up to a pressure of 2 MPa and after this pressure, follow the theoretical curve for $n = 1$. The least squares best fit line for this condition gives an average value of $n = 1.068$ which is close to the isothermal compression of air value ($n = 1$). The experimental results suggest that the value of the polytropic index must be changing over the pressure range as evidenced by the move from the $n = 1.4$ curve at low pressures to the $n = 1$ curve at the higher pressures. For higher values of X_0 , the experimental results deviate somewhat, but follow a constant polytropic index more closely.

Experimental Results of the Distributed Air Case

In this case of the experimental procedure, air was added continuously into the circuit through a venture orifice for a specific amount of time. As a result, small air bubbles of different sizes were generated which more closely represents a hydraulic system in practice. Unlike the lumped air case, since each air bubble was in contact with the surrounded oil, the total contact area of the air and oil was increased. Hence, the possibility of air dissolving into the oil was also expected to increase.

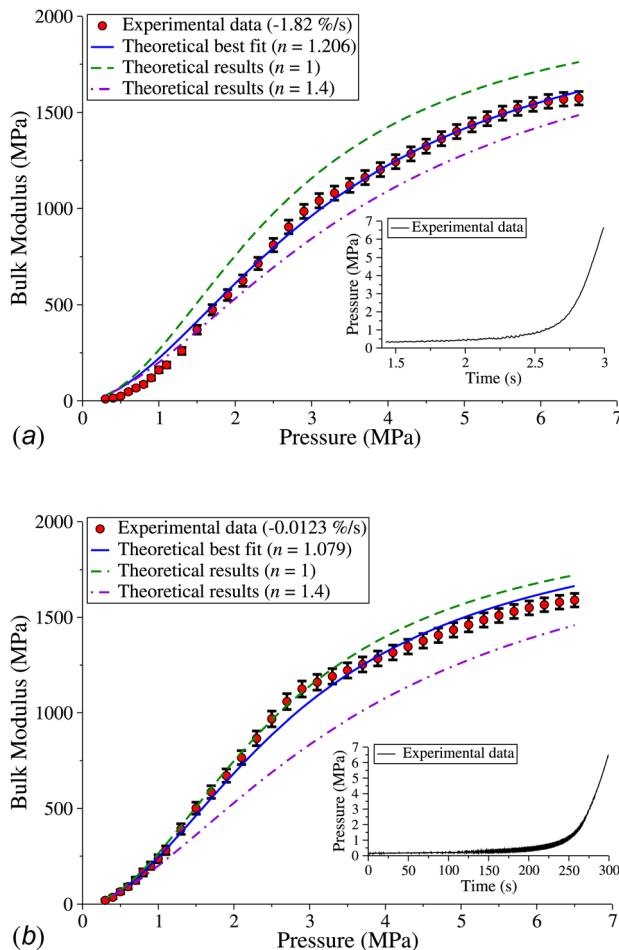


Fig. 8 (a) and (b) Nonlinear least squares curve fit of the experimental data with 3% of lumped air: (a) $K_I(P_0, T) = 1972$ MPa, $n = 1.206$, and $E = 34$ MPa; (b) $K_I(P_0, T) = 1920$ MPa, $n = 1.079$, and $E = 45$ MPa

Because of this increased possibility of air dissolving into the oil, the “compression and dissolve” model was chosen to be compared with the experimental results of this case. The same nonlinear least squares method which was explained in the baseline case was also employed here. With the same method that was explained in the baseline and lumped air cases, the parameters m and X_0 were determined. Therefore, the five unknown parameters which needed to be estimated using the least squares method were: n_1 , n_2 , $K_I(P_0, T)$, P_C , and $(X_0)_C$.

The upper and lower limits of parameters $K_I(P_0, T)$, n_1 , and n_2 were determined using the same method as explained earlier. Note that n_1 and n_2 are defined as the polytropic index of air before and after what has been defined in this work as a “saturation point,” respectively. This is a “critical point” in which it appears that air cannot be dissolved into solution due to local boundary resistance which occurs when a fluid becomes saturated. Depending on the amount of air present, parameter P_C can take on a value within a range from 0.1 MPa up to the maximum pressure of the system. Parameter $(X_0)_C$ can also range from 0 to the maximum volumetric fraction of air.

In the “Experimental Results of the Distributed Air Case for a Rapid Volume Change Rate”, “Experimental Results of the Distributed Air Case for an Intermediate Volume Change Rate”, and “Experimental Results of the Distributed Air Case for a Slow Volume Change Rate” sections, the experimental data and the theoretical best fit of each experiment are plotted based on the results of the least squares method. The estimated values of n_1 , n_2 , $K_I(P_0, T)$, P_C , and $(X_0)_C$ are also given for each experiment.

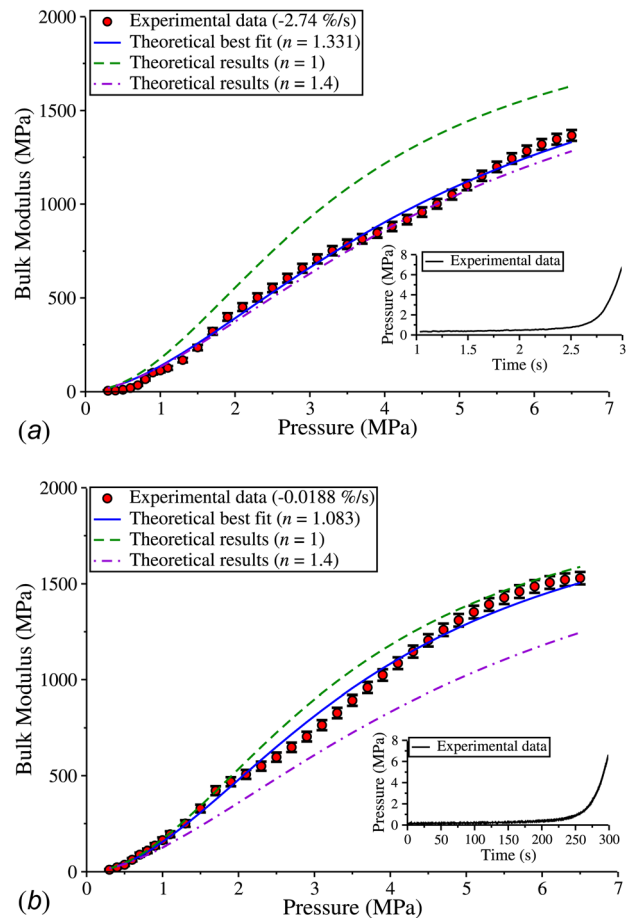


Fig. 9 (a) and (b) Nonlinear least squares curve fit of the experimental data with 5% of lumped air: (a) $K_I(P_0, T) = 1972$ MPa, $n = 1.331$, and $E = 30$ MPa; (b) $K_I(P_0, T) = 1930$ MPa, $n = 1.083$, and $E = 39$ MPa

Table 3 Summary of the experimental results in the lumped air case

	Volume change rate (%/s)	n	$K_I(P_0, T)$ (MPa)	E (MPa)
$X_0 = 1\%$	-0.659	1.068	1972	50
	-0.0045	1	1898	27
$X_0 = 3\%$	-1.82	1.206	1972	34
	-0.0123	1.079	1920	45
$X_0 = 5\%$	-2.74	1.331	1972	30
	-0.0114	1.083	1930	39

Another two sets of the theoretical results were plotted for the two extreme cases of $n = 1$ and $n = 1.4$. It is noted that these two sets were plotted based on the “compression only” model and presented only for comparison purposes. A plot of the change in pressure with time is also inserted in each figure.

Experimental Results of the Distributed Air Case for a Rapid Volume Change Rate. Figures 10(a)–10(c) depict the experimental and theoretical bulk modulus values as a function of pressure for $X_0 = 1.5\%$, 3.35% and 4.5% of distributed air when there was a rapid volume change rate. In this series of tests, it was not evident where P_C occurred. For the least squares method, an upper and lower limit of P_C was needed in order to find the theoretical bulk modulus best fit curves. In this case, the upper limit of P_C was chosen to be 6.5 MPa which corresponded to the maximum pressure range of the experiments and the lower limit was

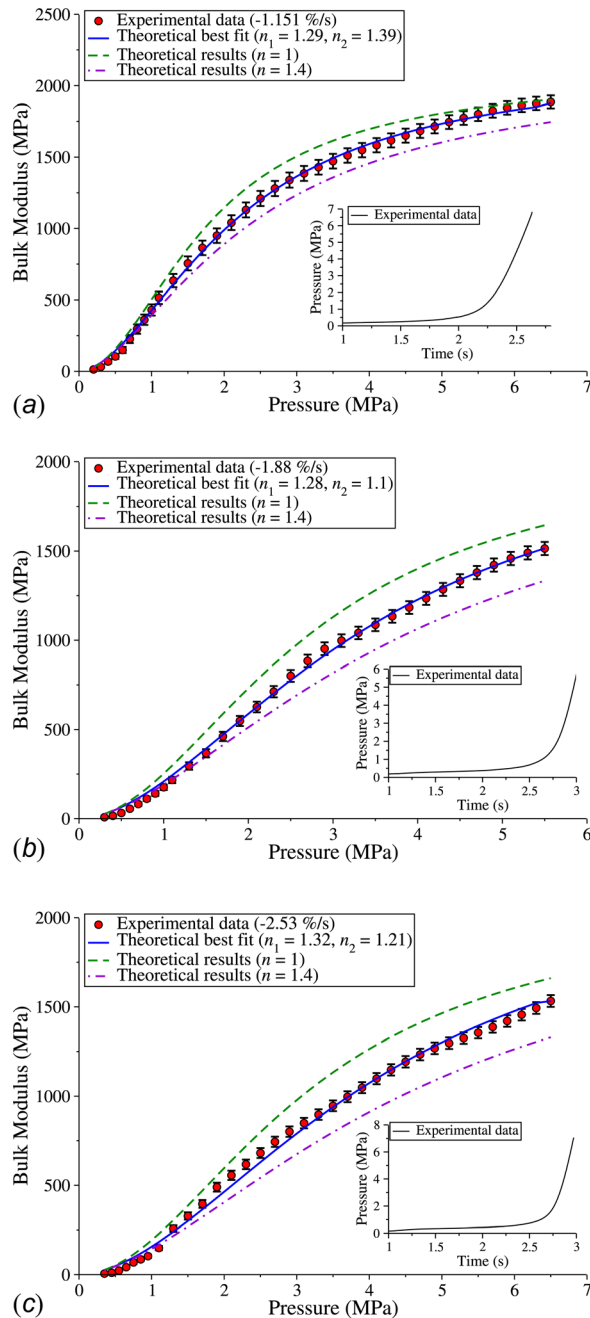


Fig. 10 (a)–(c) Nonlinear least squares curve fit of the experimental data for different amounts of distributed air for a rapid volume change rate: (a) $K_I(P_0, T) = 1972$ MPa, $X_0 = 1.5\%$, $P_C = 6.5$ MPa, $(X_0)_C = 0.97\%$, and $E = 20$ MPa; (b) $K_I(P_0, T) = 1970$ MPa, $X_0 = 3.35\%$, $P_C = 6.5$ MPa, $(X_0)_C = 2.49\%$, and $E = 20$ MPa; (c) $K_I(P_0, T) = 1970$ MPa, $X_0 = 4.5\%$, $P_C = 6.5$ MPa, $(X_0)_C = 3\%$, and $E = 31$ MPa

set to be zero. Because the least squares method in this case chose P_C to be the upper limit, it may be concluded that the real value of P_C was not reached or even was beyond the maximum pressure range of the experiments.

Since $(X_0)_C$ gives the amount of air which remains after the critical pressure P_C is reached, the real values of $(X_0)_C$ may also be different than the actual values because the real values of P_C are not known. Since the fit of the results were good, the theoretical values of P_C and $(X_0)_C$ are good approximations of the actual values.

An analysis of Figures 10(a)–10(c) shows that the experimental results always lie between the two extreme cases of $n = 1$ and $n = 1.4$. This may be a result of little or no dissolving of air into

the oil. This was confirmed by comparison of the experimental results with the “compression only” model which assumes no dissolving occurs.

Experimental Results of the Distributed Air Case for an Intermediate Volume Change Rate. Figures 11(a)–11(c) represent the experimental and theoretical bulk modulus changes as a function of pressure for 1.9%, 3.45% and 4.4% of distributed air when the volume change rate was intermediate. The volume change rate in these cases was considered intermediate, since the total

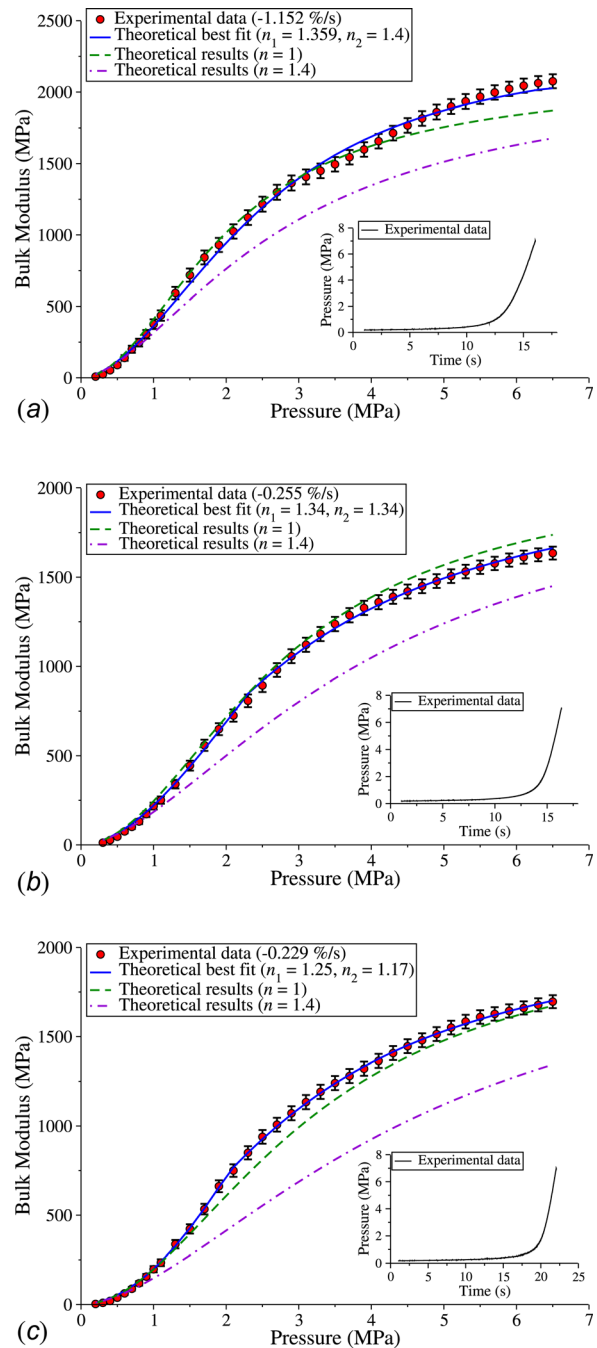


Fig. 11 (a)–(c) Nonlinear least squares curve fit of the experimental data for different amounts of distributed air for an intermediate volume change rate: (a) $K_I(P_0, T) = 1972$ MPa, $X_0 = 1.9\%$, $P_C = 6.4$ MPa, $(X_0)_C = 0.1\%$, and $E = 36$ MPa; (b) $K_I(P_0, T) = 1970$ MPa, $X_0 = 3.45\%$, $P_C = 2.2$ MPa, $(X_0)_C = 2.14\%$, and $E = 16$ MPa; (c) $K_I(P_0, T) = 1972$ MPa, $X_0 = 4.4\%$, $P_C = 2.1$ MPa, $(X_0)_C = 2.24\%$, and $E = 9$ MPa

duration of the compression time was approximately 15 s. Figures 11(a)–11(c) show that even though the compression time was relatively rapid, some dissolving of air into the oil occurred during compression. In Fig. 11(a), when the amount of distributed air was less than 2%, the critical pressure value was $P_C = 6.4$ MPa, which is close to the maximum pressure limit of the experiments.

As the amount of air was increased to 3.45%, the critical pressure value decreased to $P_C = 2.2$ MPa with an $(X_0)_C$ value of 2.14%, indicating that 2.14% of the distributed air was compressed only after the critical pressure point (no dissolving took place after this point). Increasing the amount of air to 4.4% resulted in more dissolving of air into the oil, with $P_C = 2.1$ MPa and $(X_0)_C = 2.24\%$.

Experimental Results of the Distributed Air Case for a Slow Volume Change Rate. Figure 12(a) depicts the experimental and theoretical bulk modulus changes as a function of pressure for 1.9% of distributed air when the volume change rate was $-0.00442\%/s$. The volume change rate was considered to be slow, since the total duration of the compression time was 550 s. Figure 12(a) shows that the critical pressure value was up to the maximum pressure range of the experiments. This suggests that the real value of P_C was not reached and was beyond the maximum pressure of 6.5 MPa. As a result, the real value of $(X_0)_C$ was not known, but the assumed value deemed to be a good approximation because of the fit of the experimental results to the theoretical curve.

It is also evident that after a pressure of 5 MPa was reached, the experimental results lie above the theoretical results $n = 1$. This suggests the possibility of air dissolving into the oil which could only be captured by the “compression and dissolve” model as shown by the theoretical best fit curve in the figure.

Figure 12(b) represents the experimental and theoretical bulk modulus changes as a function of pressure for 3.42% of distributed air when the volume change rate was $-0.00618\%/s$. Since dissolving of air occurred, the experimental results moved above the $n = 1$ trace as pressure increased. P_C occurred at 1 MPa and the remaining percentage of air left in the oil was $(X_0)_C = 2.22\%$. After this point, the results followed a “compression only” curve indicating that perhaps saturation of air in the oil had occurred.

Figure 12(c) represents the experimental and theoretical bulk modulus changes as a function of pressure for 4.4% of distributed air when the volume change rate was $-0.0115\%/s$. Again in this experiment, the dissolving effect of air in the oil moved all of the experimental results above $n = 1$ curve. The critical pressure was $P_C = 0.8$ MPa and saturation occurred at a point where the remaining percentage of air was $(X_0)_C = 2.12\%$. It should be noted that even though the initial percentage of air was 4.4%, the remaining percentage of air when saturation occurred, $(X_0)_C$, was nearly the same as when the initial percentage of air was 3.42%.

Discussions and Conclusions

In this paper, all of the experimental results for three different cases of the experimental procedure (baseline, lumped and distributed air) were presented and compared to their appropriate theoretical models. The unknown parameters in each model were determined using a nonlinear least squares method.

In the baseline case, the isothermal and adiabatic tangent bulk modulus values of pure oil were estimated. These results were compared with existing isothermal and adiabatic values from the literature and it was found that there was good agreement between the values. This confirmed the validity of the baseline case as well as the experimental setup. The bulk modulus of the pure test oil was also determined when different volume change rates were considered. The values of $K_I(P_0, T)$ versus oil compression time were summarized in a table and used in the lumped and distributed air cases to determine the lower and upper limits of the test oil bulk modulus which was needed in the least squares method.

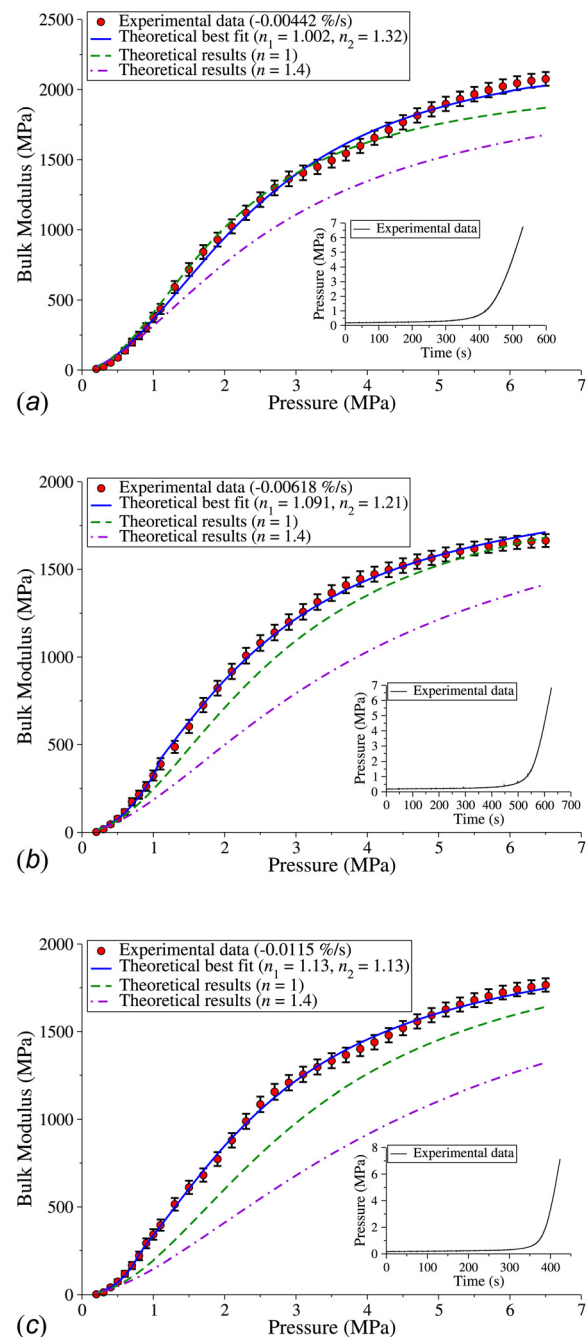


Fig. 12 (a)–(c) Nonlinear least squares curve fit of the experimental data for different amounts of distributed air for a slow volume change rate: (a) $K_I(P_0, T) = 1900$ MPa, $X_0 = 1.9\%$, $P_C = 6.1$ MPa, $(X_0)_C = 1.09\%$, and $E = 37$ MPa; (b) $K_I(P_0, T) = 1890$ MPa, $X_0 = 3.42\%$, $P_C = 1$ MPa, $(X_0)_C = 2.22\%$, and $E = 18$ MPa; (c) $K_I(P_0, T) = 1932$ MPa, $X_0 = 4.4\%$, $P_C = 0.8$ MPa, $(X_0)_C = 2.12\%$, and $E = 18$ MPa

Experimental results of the lumped air case were presented for three different amounts of air (1%, 3%, and 5%) at different volume change rates. It was found that regardless of the amount of air or the volume change rate, the experimental results agreed well with the “compression only” model, suggesting that an insignificant amount of air was dissolved in the oil during that case. There have been contradicting ideas in the literature regarding the isothermal or adiabatic nature of compression of air in the oil. Hayward [17] reported that when a column of bubbly oil was compressed rapidly, air bubbles followed a nearly isothermal compression ($n = 1$). This result was in contradiction with the

conclusions of Yu et al. [6] who suggested that the air compression was adiabatic ($n = 1.4$). The main reason for such a divergence of opinions may be explained by the fact that the true value of the polytropic index “ n ” is influenced by many factors such as the volume change rate, the rate of heat transfer from the air to the surrounding environment, and the dissolving of air into the oil. The experimental results showed some variations in the value of “ n ” over the pressure range, indicating that the polytropic index “ n ” is not a constant and should be treated as a variable. Some deviations of the experimental results from the theoretical models (especially in the lumped air case) have resulted more probably due to treating the polytropic index “ n ” as a constant. The theoretical models could be improved by analyzing and modeling the heat transfer to the surroundings during the compression.

Experimental results of the distributed air case were also presented for different amounts of air (approximately 1.5%, 3.5%, and 4.5%) at different volume change rates. The “compression and dissolve” model was chosen as a fit for comparison with the experimental results due to the higher possibility that air would be dissolved into the oil. The experimental results were also compared with the two extreme cases of the “compression only” model ($n = 1$ and $n = 1.4$). A comparison of the experimental results with the “compression only” model gave a good insight whether any significant dissolving of air occurred.

The maximum amount of air that can be dissolved in the oil depends on many factors, such as type of the oil, type of the gas (here air is used), and temperature, thus needs to be determined experimentally. This maximum amount is shown as $(X_0)_C$ in this paper. The critical pressure (P_C) will occur on the curve of $(X_0)_C$ and its value is highly dependent on the volume change rate and also on the volumetric fraction of air at atmospheric pressure (X_0). From the experimental results of the slow volume change rate, $(X_0)_C = 2.2\%$ was approximately determined.

For the experiments in which $X_0 < (X_0)_C$, it is clear that even by dissolving all the air in the oil, the saturation limit of the oil would never be reached. Therefore, the critical pressure would be expected to happen at higher pressures. Figures 10(a), 11(a), and 12(a), show the results of $X_0 < (X_0)_C$ for rapid, intermediate, and slow volume change rates respectively. The value of P_C at all of the figures, regardless of the volume change rate, is close to the maximum pressure of the experimental system ($P_C = 6.5$ MPa). It is expected that $(X_0)_C$ would never happen and the estimated value of $(X_0)_C$ should show a value near zero. However, this is not the case for the estimated values of $(X_0)_C$. This is due to the maximum pressure limit of the experimental setup and lack of enough experimental results after $P_C = 6.5$ MPa. Since the maximum limit of the experimental setup was 6.5 MPa and P_C was estimated near this value, it might conclude that the real value of P_C was not reached and even was beyond the maximum pressure range of the experiments. It is suggested that before estimating the parameters, simplify the model by setting the value of $(X_0)_C$ to zero. This would decrease the fitting error and result in more accurate estimates.

If $(X_0) > (X_0)_C$, depending on the volume change rate, two conditions may happen:

- (1) Rapid volume change: Since the volume change rate was rapid, there may not be enough time for the air to saturate the oil. Figures 10(b) and 10(c) show the results of $X_0 > (X_0)_C$ for rapid volume change rate. The air will continue to be dissolved till it reaches to $(X_0)_C$. Since the volume change rate was fast, the saturation occurred at higher pressures as predicted by the least square method at $P_C = 6.5$ MPa. Due to the lack of the experimental data after $P_C = 6.5$ MPa, the estimated values of P_C and $(X_0)_C$ may not represent the accurate values of these parameters.
- (2) Slow volume change: Since the volume change rate was slow, there would be enough time for the air to saturate the oil. Figures 12(b) and 12(c) show the results of $X_0 > (X_0)_C$ for slow volume change rate. It was observed that $(X_0)_C$

was reached at lower critical pressure values, due to the slow volume change. The accurate value of $(X_0)_C$ could be estimated in this case, since there is enough experimental data after P_C . The value of $(X_0)_C$ was estimated to be approximately $(X_0)_C = 2.2\%$.

Determining the interface conditions between air and the oil as well as the volume change rate could be helpful in choosing the suitable model of the effective fluid bulk modulus. It should be noted that these conclusions are primarily based on results in which the pressure range is 0 to 6.9 MPa, X_0 range is from 1% to 5% and a constant temperature of 24 °C is maintained.

If the only contact between oil and air is on the top of the oil column (lumped air case), the experimental results showed that regardless of the volume change rate, the “compression only” model could be successfully used.

If air is added in the form of small air bubbles with different sizes, the “compression and dissolve” model should be used. However, the values of P_C and $(X_0)_C$ need to be estimated using experimental measurements.

The “compression and dissolve” model developed in this paper could be extended to include a mathematical equation in which P_C is related to the volume change rate and size and distribution of the air bubbles. Definitely, there would be unique values of parameters associated with any new experiment and the presented model was a step forward in including the dissolving effect of air into the effective fluid bulk modulus and an experimental method to find these parameters.

Nomenclature

- E = average error (MPa)
- K_{ec} = “compression only” effective bulk modulus model (MPa)
- K_{ecd} = “compression and dissolve” effective bulk modulus model (MPa)
- K_{eff} = tangent effective bulk modulus (MPa)
- K_g = tangent bulk modulus of the air (MPa)
- K_l = tangent bulk modulus of the oil (MPa)
- $\hat{K}_{eff}$ = experimental value of K_{eff} (MPa)
- m = slope of the pure oil bulk modulus versus pressure curve
- n = polytropic index of air
- n_1 = polytropic index of air before critical pressure
- n_2 = polytropic index of air after critical pressure point
- P = instantaneous pressure (absolute) (MPa)
- P_g = instantaneous air pressure (MPa)
- P_C = critical pressure (absolute) (MPa)
- P_0 = atmospheric pressure (absolute) (MPa)
- T = instantaneous temperature (K)
- T_0 = initial temperature (K)
- V = volume of fluid (entrained air + oil) at P and T (m^3)
- V_g = volume of entrained air at P and T (m^3)
- V_{gc} = instantaneous volume of entrained air due to compression (m^3)
- V_{gcd} = instantaneous volume of entrained air due to both compression and dissolve (m^3)
- V_l = volume of oil at P and T (m^3)
- V_0 = volume of fluid (entrained air + oil) at P_0 and T_0 (m^3)
- V_{g0} = volume of entrained air at P_0 and T_0 (m^3)
- V_{l0} = volume of oil at P_0 and T_0 (m^3)
- X_0 = volumetric fraction of entrained air at P_0 and T_0
- $(X_0)_C$ = critical volumetric fraction of entrained air. It is defined as the volumetric fraction of entrained air at atmospheric pressure and temperature of interest which is related to the critical “compression only” bulk modulus curve
- Z = optimization matrix
- Z_l = Z lower limit
- Z_u = Z upper limit
- β = volume fraction of air present in the baseline case
- ε_p = dimensionless uncertainty of the pressure measurement
- ε_{V_0} = dimensionless uncertainty of the volume measurement

ε_x = dimensionless uncertainty of the displacement measurement
 θ = mass fraction of entrained air due to dissolving

Appendix A: Deriving the Effective Bulk Modulus Model for a Mixture of Air and Oil

Applying the fundamental definition of bulk modulus, the true effective bulk modulus equation for a mixture of oil and air is

$$K_{\text{eff}} = -(V_{\text{gc}} + V_1) \left(\frac{dP}{dV_1 + dV_{\text{gc}}} \right) \quad (\text{A1})$$

In Eq. (A1), the change in the volume of entrained air is considered to be only due to compression of entrained air. In reality, however, an additional volume of entrained air is lost due to air dissolving into the oil. This additional volume decrease of entrained air is not included in Eq. (A1). Basically, Eq. (A1) is the same equation which was used in the “compression only” bulk modulus model.

The challenge then becomes one of how the effective bulk modulus of the mixture of oil and air can be modeled when the volume of air decreases, not only because of compression but also because of air dissolving into the oil, knowing that the additional volume decrease cannot be directly included in Eq. (A1). The reader is referred to Ref. [9] for a more detailed discussion. Equation (A1) is written in another mathematical form as

$$\frac{1}{K_{\text{eff}}} = -\frac{1}{V_1 + V_{\text{gc}}} \left(\frac{dV_1}{dP} \frac{V_1}{V_1} + \frac{dV_{\text{gc}}}{dP} \frac{V_{\text{gc}}}{V_{\text{gc}}} \right) \quad (\text{A2})$$

Using the oil and air bulk modulus definitions, Eq. (A2) simplifies to

$$\frac{1}{K_{\text{eff}}} = \frac{1}{V_1 + V_{\text{gc}}} \left(\frac{V_1}{K_1} + \frac{V_{\text{gc}}}{K_g} \right) \quad (\text{A3})$$

Equation (A3) is a useful equation for finding the effective bulk modulus when both the effect of compression and the effect of dissolving are considered; however, some approximations are necessary as will be discussed here.

Consider Fig. 13. For very small changes in pressure, the corresponding change in volume of entrained air is shown. V_{g0} is the initial volume of entrained air at pressure P_0 (represented as point A). When the pressure increases from P_0 to P_1 , the volume of entrained air decreases according to the ideal gas law and reduces to V_{gc1} (represented as point B). As soon as the pressure reaches P_1 , some of the entrained air is dissolved into the oil and the volume of entrained air decreases to V_{gcd1} (represented as point C).

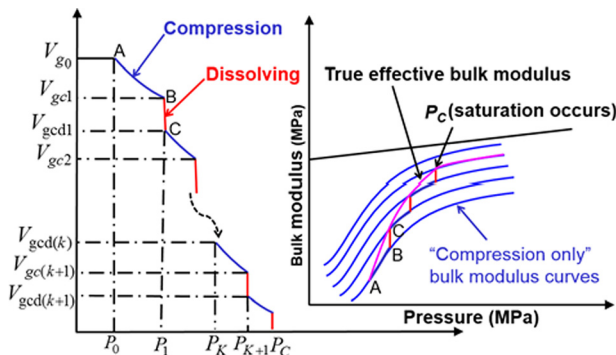


Fig. 13 Graphical representation showing how to use “compression only” bulk modulus curves in order to find “compression and dissolve” model

The insert of Fig. 13 shows how the bulk modulus value traces across the corresponding “compression only” bulk modulus curves.

The series of “compression only” bulk modulus curves at a different X_0 are drawn in order to assist in the understanding of how the true effective bulk modulus value is obtained. Since the trace of point A to point B is on one of the “compression only” curves, the corresponding bulk modulus can be obtained using the model

$$\frac{1}{K_{\text{eff}}} = \frac{1}{V_1 + V_{\text{gc}}} \left(\frac{V_1}{K_1} + \frac{V_{\text{gc}}}{K_g} \right) \quad (\text{A4})$$

At point B, the volume of entrained air decreases due to air dissolving into the oil and the trace will jump from point B to point C. The bulk modulus at point C will still be on one of the “compression only” curves but on a curve with less amount of air. Knowing the amount of entrained air at point C, which is V_{gcd1} , the effective bulk modulus at point C can be found by

$$\frac{1}{K_{\text{eff}}} = \frac{1}{V_1 + V_{gcd1}} \left(\frac{V_1}{K_1} + \frac{V_{gcd1}}{K_g} \right) \quad (\text{A5})$$

If it is assumed that the pressure increase from P_0 to P_1 is slow enough and the mixture reaches a thermodynamic equilibrium state, the true effective bulk modulus will follow the curve AC on the bulk modulus versus pressure plot. Each point on this curve represents the “compression only” bulk modulus value corresponding to the volume of entrained air at that point.

In Ref. [9], the authors assumed that as pressure keeps on increasing, there will be a point where there will be no more entrained air and this point was defined as the critical pressure point (P_c). A model was developed in Ref. [9] to account for this, but the model did not follow the experimental results very well. Experimental results of Refs. [8] and [9] indicated that even at higher pressures there was some air that did not completely dissolve into the oil; thus, it was necessary to re-examine those results in greater detail and improve that model.

As was shown in Fig. 13, a series of “compression only” bulk modulus curves were used to indicate how the effective bulk modulus increases by crossing through the “compression only” bulk modulus curves. This technique was used to examine the results from Ref. [9]. Figure 14 shows the trace of the experimental results plotted on top of a series of “compression only” curves with different amounts of air. It can be seen that as pressure increases, the effective bulk modulus trace passes across the “compression only” curves due to the loss of entrained air which is being dissolved into the oil. This trend continues until it reaches

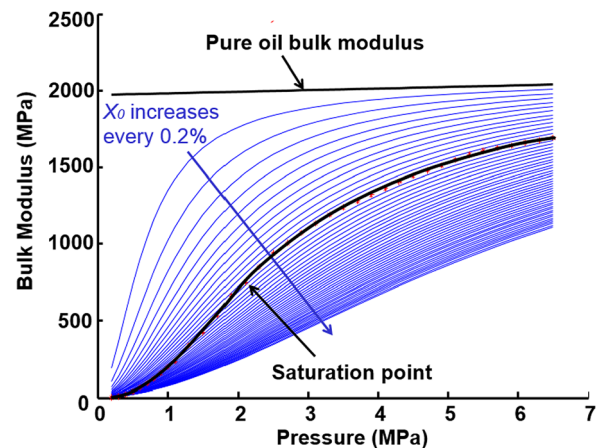


Fig. 14 Comparison of the experimental results with a series of “compression only” bulk modulus curves for isothermal process

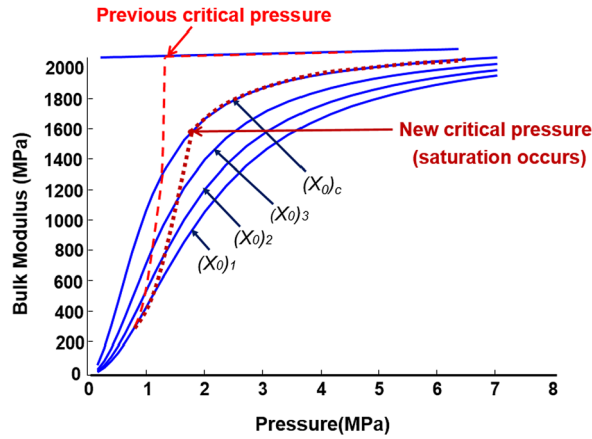


Fig. 15 A new critical pressure definition is introduced based on the saturation limit of oil

a point labeled as saturation point. After this point, no significant “jumping” through the “compression only” curves are observed; indeed, the experimental results tend to follow one of the “compression only” curves. This indicates that no significant dissolving occurs after this saturation point, and the remaining air can be considered to be only compressed.

The reason for this behavior may be explained by the fact that there is a practical limit to Henry’s law in which the air cannot be dissolved into the oil after it has reached a particular saturation limit. After this saturation limit occurs, any air that remains in the oil will not be dissolved or very little will be dissolved [18]. Reference [18] reported that this practical limit was found to be within 1.3–1.7 MPa when nitrogen gas is dissolved in MIL-H-5606 oil. Moreover, experimental findings of Ref. [17] showed that by compressing a column of a mixture of oil and air, initially the air bubbles dissolve rapidly but tend to slow down as the skin of oil around each bubble is saturated with dissolved air.

Consequently, the definition for the critical pressure, where it is assumed that all the air is dissolved into the oil, needs to be re-addressed according to the saturation limit of oil. Figure 15 shows the new definition of the critical pressure.

In the new definition, after the critical pressure is reached, the effective bulk modulus will follow the “compression only” curve with known “ X_0 ” which here is called $(X_0)_c$. Note that $(X_0)_1$ is the same as the volumetric fraction of air at atmospheric pressure which was already defined by X_0 . Therefore, the effective bulk modulus at each point below the critical pressure point can be estimated by

$$\frac{1}{K_{\text{ecd}}} = \frac{V_{\text{gcd}}}{V} \frac{1}{K_g} + \frac{V_1}{V} \frac{1}{K_1} \quad (\text{A6})$$

According to the new definition of critical pressure, when the critical pressure is reached, the oil is saturated and no more air will be dissolved in the oil. After this critical point, the remaining air will tend to follow a “compression only” bulk modulus curve related to the critical volumetric fraction of air $(X_0)_c$. Note that the polytropic index of air was considered to be different before and after the saturation point. n_1 is the polytropic index of air before the saturation point and n_2 is after the saturation point.

The volume of entrained air in oil when pressure is less than the critical pressure is given by

$$V_{\text{gcd}}(P, T) = \left(\frac{P_0}{P}\right)^{\frac{1}{n_1}} \frac{T}{T_0} (X_0) \theta \quad P < P_C \quad (\text{A7})$$

From Fig. 16, the volumetric fraction of air (θ) at $P < P_C$ is found by

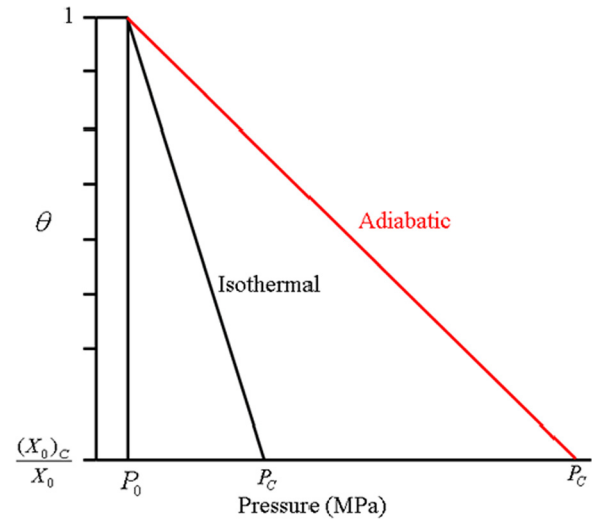


Fig. 16 Mass fraction of entrained air due to dissolving which is based on the new definition for the critical pressure

$$\theta = \left(\frac{P_C - P}{P_C - P_0}\right) \left(1 - \frac{(X_0)_C}{(X_0)}\right) + \frac{(X_0)_C}{(X_0)} \quad (\text{A8})$$

Figure 16 shows how θ is different for isothermal and adiabatic process. In an isothermal process, when the process is assumed to be very slow, equilibrium conditions exist between the air bubbles and the oil and Henry’s law can be used to calculate the amount of air dissolved in the oil. The linear relationship shown in Fig. 16 between θ and pressure was derived based on the application of the Henry’s law. The pressure at which no more air is dissolved in the air is P_C .

In adiabatic conditions, the mixture of oil and air is compressed quickly and equilibrium conditions between the air bubbles and oil may not be reached. In this case, Henry’s law cannot be directly applied. However, it can be assumed that θ follows a linear relationship with pressure where P_C is reached at much higher pressure than the isothermal process.

For pressures equal to or higher than the critical pressure, the remaining air will not be dissolved and will follow the compression rule

$$V_{\text{gcd}}(P, T) = \left(\frac{P_0}{P}\right)^{\frac{1}{n_2}} \frac{T}{T_0} (X_0)_C \quad P \geq P_C \quad (\text{A9})$$

Combining Eqs. (A6)–(A9), a new final model which relates the effective bulk modulus to the volumetric variation of the air due to both compression and dissolving of air in the oil, and also the change in volume and bulk modulus of pure oil to the pressure and temperature is given by

$$\left\{ \begin{array}{l} K_{\text{ecd}} = \frac{V_1(P, T) + V_{\text{gcd}}(P, T)}{\frac{V_1(P, T)}{K_1(P, T)} + \frac{1}{K_{g1}} V_{\text{gcd}}(P, T)} \quad P \leq P_C \\ K_{\text{ecd}} = \frac{V_1(P, T) + \left(\frac{P_0}{P}\right)^{\frac{1}{n_2}} \frac{T}{T_0} (X_0)_C}{\frac{V_1(P, T)}{K_1(P, T)} + \frac{1}{K_{g2}} \left(\frac{P_0}{P}\right)^{\frac{1}{n_2}} \frac{T}{T_0} (X_0)_C} \quad P > P_C \end{array} \right. \quad (\text{A10})$$

where

$$K_{g1} = n_1 P$$

$$K_{g_2} = n_2 P$$

$$V_{gcd}(P, T) = \left(\frac{P_0}{P}\right)^{\frac{1}{n_1}} \frac{T}{T_0} X_0 \left(\left(\frac{P_C - P}{P_C - P_0} \right) \left(1 - \frac{(X_0)_C}{X_0} \right) + \frac{(X_0)_C}{X_0} \right)$$

$$K_1(P, T) = K_1(P_0, T) + m(P - P_0)$$

and

$$V_1(P, T) = V_1(P_0, T) \left(1 + \frac{m}{K_1(P_0, T)} (P - P_0) \right)^{-\frac{1}{m}}$$

Appendix B: Isothermal and Adiabatic Bulk Modulus Estimation of “Esso Nuto H68” Oil

The objective of the Appendix B is to estimate the isothermal and adiabatic bulk modulus (secant and tangent) of “Esso Nuto H68” oil using experimental relations developed in Ref. [19].

In the relationships provided in Ref. [19], the isothermal and adiabatic secant bulk modulus of any normal hydraulic oil is predicted within about 5% with only knowing the kinematic viscosity of oil at atmospheric pressure and 20 °C. The kinematic viscosity of “Esso Nuto H68” oil was provided by the manufacturer at two temperatures of 40 °C and 100 °C (Table 4). In order to obtain the kinematic viscosity value of the oil at 20 °C, the relationships given by ASTM standard (ASTM-D341-09) is used [20]. The formulations provided by the ASTM standard, allows determining the kinematic viscosity of any mineral oil over the temperature range of −70 to 370 °C, only by knowing two kinematic viscosity–temperature points.

Figure 17 shows a plot of kinematic viscosity versus temperature (at atmospheric pressure) for Esso Nuto H68 oil which was obtained using the ASTM standard relationships. From this plot, the kinematic viscosity of the oil at 20 °C is found to be 219 cS.

After obtaining the kinematic viscosity of the oil at 20 °C and atmospheric pressure, the equations provided in Ref. [19] were used to obtain the isothermal and adiabatic secant bulk modulus variation of “Esso Nuto H68” oil with pressure. These equations were determined as

Table 4 Viscosity of test fluid (Esso Nuto H68) at 40 °C and 100 °C provided by the manufacturer

Temperature (°C)	Viscosity (cSt)
40	68
100	8.5

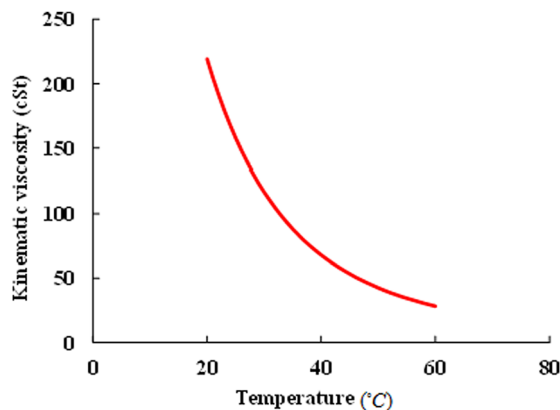


Fig. 17 Viscosity variation of Esso Nuto H68 with temperature

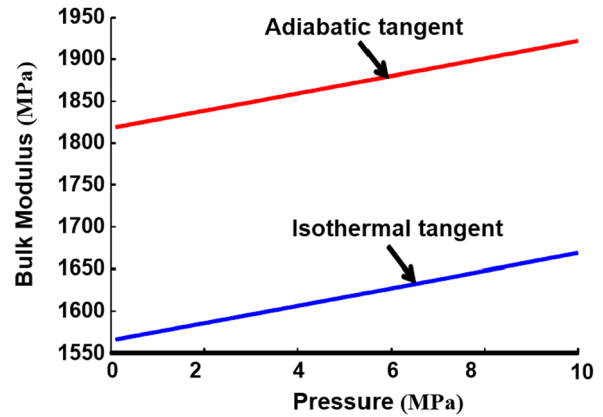


Fig. 18 Isothermal and adiabatic tangent bulk modulus variation of Esso Nuto H68 oil with pressure at a temperature of 24 °C

$$\bar{K}_T(P, T) = \frac{K_T(P_0, T)}{10} + 5.6P_g \quad (B1)$$

$$K_T(P_0, T) = (1.3 + 0.15 \log \nu(P_{atm}, 20^\circ C)) 10^{4 + \frac{20-T}{435}}$$

and

$$\bar{K}_S(P, T) = \frac{K_S(P_0, T)}{10} + 5.6P_g \quad (B2)$$

$$K_S(P_0, T) = (1.57 + 0.15 \log \nu(P_{atm}, 20^\circ C)) 10^{4 + \frac{20-T}{417}}$$

In these equations, $\nu(P_{atm}, 20^\circ C)$ represents the kinematic viscosity of oil at atmospheric pressure and 20 °C which for the Esso Nuto H68 oil was found to be: $\nu(P_{atm}, 20^\circ C) = 219$ cS. The variation of isothermal and adiabatic secant bulk modulus of the Esso Nuto H68 oil with pressure at any temperature of interest is found using Eqs. (B1) and (B2). These predicted equations for the secant bulk modulus are converted to the tangent bulk modulus values using Eq. (B3) as

$$K(P, T) = \frac{\bar{K}(P, T)(\bar{K}(P, T) - P_g)}{K(P_0, T)} \quad (B3)$$

Figure 18 represents a plot of the isothermal and adiabatic tangent bulk modulus of the Esso Nuto H68 oil which was plotted at a temperature of 24 °C. The estimated values at a temperature of 24 °C are determined as

$$K_T(P, 24^\circ C) = 1615 + 10.4(P - P_0) \quad (B4)$$

$$K_S(P, 24^\circ C) = 1878 + 10.4(P - P_0)$$

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